

15/12/15

$$A \simeq B \iff \exists f: A \xrightarrow{\text{ενί}} B$$

αμφίμ.

$$A \simeq A$$

$$A \simeq B \iff B \simeq A$$

$$A \simeq B \wedge B \simeq \Gamma \Rightarrow \blacksquare A \simeq \Gamma$$

Θ.Σ.α. $\mathbb{N} \simeq \mathbb{N}_0$

$$f(x) = 2x, x \in \mathbb{N}$$

$$\blacksquare f(x) = f(y) \stackrel{?}{\implies} x = y$$

$$2x = 2y \implies x = y$$

$$(\forall y \in \mathbb{N}_0) (\exists x \in \mathbb{N}) f(x) = y$$

$$y \text{ ζακιν, } y \in \mathbb{N}_0 \implies y/2 \in \mathbb{N} \bullet \text{ Τότε } f(y/2) = 2 \cdot y/2 = y$$

Άρα f : ενί και αμφιμονοσήμανη άρα $f: \mathbb{N} \rightarrow \mathbb{N}_0$

Θ.Σ.α. $\mathbb{N} \simeq \mathbb{N}_n$

$$g(x) = 2x - 1$$

φίτην ρε.α. σι g αμφιμονοσήμανη και ενί.

$$(\forall y \in \mathbb{N}_n) (\exists x \in \mathbb{N}) g(x) = y$$

$$y \in \mathbb{N}_n \implies y+1 \in \mathbb{N}_n \implies \frac{y+1}{2} \in \mathbb{N}$$

$$g\left(\frac{y+1}{2}\right) = 2 \cdot \frac{y+1}{2} - 1 = y$$

$$\mathbb{Z} \subset \mathbb{N}$$

$$f(x) = \begin{cases} -2x, & x < 0 \\ 2x+1, & x \geq 0 \end{cases}$$

Ex. 5.2 $\mathbb{R} \simeq (-1, 1)$

$$f: \mathbb{R} \rightarrow (-1, 1)$$

$$x \rightarrow \frac{x}{1+|x|}$$

Contoh $f(x) = \frac{x}{1+|x|}, x \in \mathbb{R}$

$$|f(x)| = \frac{|x|}{1+|x|} < 1 \implies -1 < f(x) < 1$$

$$f(x) = f(y)$$

$$x \geq 0 \text{ dan } y \geq 0 \implies$$

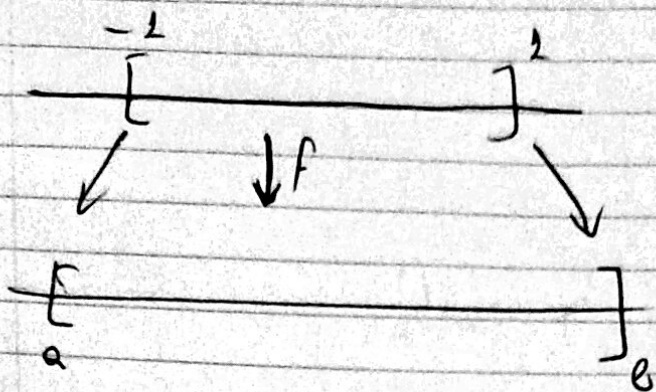
$$x < 0 \text{ dan } y < 0 \implies$$

$$\frac{x}{1+x} = \frac{y}{1+y} \implies x=y$$

$$\frac{x}{1-x} = \frac{y}{1-y} \implies x=y$$

$$\left. \begin{array}{l} x < 0 \text{ dan } y \geq 0 \\ x \geq 0 \text{ dan } y < 0 \end{array} \right\} \text{anapinzaran}$$

Apa u f anapinzaran



$$f(x) = \frac{b-a}{2}x + \frac{b+a}{2}$$

$$a < b, \quad (-L, L) \simeq (a, b)$$

$$[-L, L] \simeq [a, b]$$

$$f(x) = kx + d$$

$$f(-L) = a \Rightarrow -kL + d = a$$

$$f(L) = b \Rightarrow kL + d = b$$

$$[-L, L) \simeq [a, b)$$

$$(-L, L] \simeq (a, b]$$

Apa $\mathbb{R} \simeq (a, b), a < b$

$(-L, L), [-L, L], (-L, L], [-L, L)$ είναι όλα isomorfika

$$f(x) = \begin{cases} L/2, & x = L \\ L/3, & x = -L \\ \frac{L}{v+2}, & x = L/v, v \in \mathbb{N} \\ x, & x \in A = [-L, L] - (\{-L\} \cup \{L/v, v \in \mathbb{N}\}) \end{cases}$$

$\Theta_{\Delta_0} \quad (-\infty, a) \cong (0, L]$

$$f(x) = \frac{x-a}{L+x-a}$$